

Algoritmi numerici pentru optimizare

I - Introducere

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Formularea problemei

Să se găsească n parametri independenți, notați $x_1^*, x_2^*, \dots, x_n^*$, pentru care expresia E este minimă, unde

$$E = f(x_1, x_2, \dots, x_n), \quad (1)$$

și $f : \Omega \rightarrow \mathbb{R}$, $\Omega \subset \mathbb{R}^n$, este dată.

Pe scurt:

$$(x_1^*, x_2^*, \dots, x_n^*) = \arg \min f(x_1, x_2, \dots, x_n). \quad (2)$$

Notății

$$\mathbf{x} = [x_1, x_2, \dots, x_n]^T \in \Omega. \quad (3)$$

$$\mathbf{x}_{\min} = [x_1^*, x_2^*, \dots, x_n^*]^T \in \Omega \quad (4)$$

$$\mathbf{x}_{\min} = \arg \min f(\mathbf{x}), \quad \mathbf{x} \in \Omega$$

$$E_{\min} = f(\mathbf{x}_{\min}).$$

Formularea problemei

$$\mathbf{x}_{\min} = \arg \min f(\mathbf{x}), \quad \mathbf{x} \in \Omega \quad (5)$$

$$E_{\min} = f(\mathbf{x}_{\min}). \quad (6)$$

Observații:

1 Min / Max - limitare ?

$$\max \{ f(\mathbf{x}) \mid \mathbf{x} \in \Omega \} = - \min \{ -f(\mathbf{x}) \mid \mathbf{x} \in \Omega \}$$

2 Optimizarea "scalară" - un singur număr înglobează criterii

- de proiectare ($\|$ performanța cerută – cea obținută $\|$);
- de economie (prețul).

$\Rightarrow f$ este numită *funcție obiectiv*, *funcție de cost*, *funcție de merit*, *criteriu de performanță*.

Minime globale/locale

$$\mathbf{x}_{\min} = \arg \min f(\mathbf{x}), \quad \mathbf{x} \in \Omega \quad (7)$$

$$E_{\min} = \min \{ f(\mathbf{x}) \mid \mathbf{x} \in \Omega \} \quad (8)$$

\mathbf{x}_{\min} este *minim global* dacă

$$E_{\min} \leq f(\mathbf{x}), \quad \forall \mathbf{x} \in \Omega \quad (9)$$

- dacă $E_{\min} \leq f(\mathbf{x})$ doar într-o vecinătate a lui \mathbf{x}_{\min} atunci minimul este *local*.
- în practică este dificil de stabilit dacă un minim găsit este local sau global;
- minimul global s-ar putea să nu fie unic.

Restricții

$$\mathbf{x}_{\min} = \arg \min f(\mathbf{x}), \quad \mathbf{x} \in \Omega \quad (10)$$

$$E_{\min} = \min \{ f(\mathbf{x}) \mid \mathbf{x} \in \Omega \} \quad (11)$$

$\Omega =$ *domeniu de căutare*

- Dacă $\Omega = \mathbb{R}^n$ atunci optimizarea este **fără restricții de domeniu**
- problemele reale sunt în foarte rare cazuri fără restricții;
- analiza metodelor de optimizare fără restricții este importantă pentru
 - 1 a înțelege principiile de bază ale optimizării cu restricții;
 - 2 a reformula (dacă este posibil) problemele cu restricții ca probleme fără restricții.

Restricții

Tipuri de restricții

- *de domeniu*

$$x_{L,i} \leq x_i \leq x_{U,i} \quad (12)$$

unde $x_{L,i}$ și $x_{U,i}$ sunt limite fixate, $i = 1, \dots, n$;

- *de tip inegalitate*

$$g_i(x_1, x_2, \dots, x_n) \leq 0 \quad (13)$$

unde $g_i : \Omega \rightarrow \mathbb{R}$, $i = 1, \dots, m$ sunt m funcții date.

- *de tip egalitate*

$$h_j(x_1, x_2, \dots, x_n) = 0 \quad (14)$$

unde $h_j : \Omega \rightarrow \mathbb{R}$, $j = 1, \dots, p$ sunt p funcții date.

Obs: restricțiile de domeniu pot fi reformulate ca restricții de tip inegalitate.

Forma generală a problemei minimizării cu restricții

$$\mathbf{x} =? \quad \min\{f(\mathbf{x}) : \mathbf{x} \in \Omega, g_i(\mathbf{x}) \leq 0, i \in \mathcal{I}; h_j(\mathbf{x}) = 0, j \in \mathcal{J}\} =?, \quad (15)$$

unde $\Omega \subset \mathbb{R}^n$, \mathcal{I} și \mathcal{J} sunt mulțimi de indici.

- Domeniul de căutare în care restricțiile sunt satisfăcute = *domeniu admisibil*;
- Optimizarea cu restricții este mult mai dificilă decât optimizarea fără restricții.

Clasificarea problemelor de optimizare scalară

$$\min\{f(\mathbf{x}) : \mathbf{x} \in \Omega \subset \mathbb{R}^n, g_i(\mathbf{x}) \leq 0, i = 1, m; h_j(\mathbf{x}) = 0, j = 1, p\}$$

- 1 Probleme fără restricții: $\Omega = \mathbb{R}^n, m = 0, p = 0$;
- 2 Probleme doar cu restricții de domeniu: $m = 0, p = 0$;
- 3 Probleme de programare¹ neliniară: f, g_i, h_j neliniare;
- 4 Probleme de programare liniară: f, g_i, h_j liniare;
- 5 Probleme de programare pătratică: f pătratică; g_i, h_j liniare;
- 6 Probleme de optimizare a rețelelor (f, g_i, h_j provin din analiză de grafuri);
- 7 Programare întreață ($\mathbf{x} \in \mathbb{Z}^n$);
- 8 Programare mixtă (unii parametri sunt întregi, iar alții sunt reali);

¹"programare" = optimizare

Formularea problemei optimizării vectoriale

Urmăresc satisfacerea simultană a mai multor obiective (e.g.: cost minim, randament maxim, solicitări minime, etc.).

$$\min\{\mathbf{F}(\mathbf{x}) \text{ ; } g_i(\mathbf{x}) \leq 0, i = 1, m; h_j(\mathbf{x}) = 0, j = 1, p\} \quad (16)$$

unde $\mathbf{F} : \Omega \rightarrow \mathbb{R}^q, \Omega \subset \mathbb{R}^n, g_i : \Omega \rightarrow \mathbb{R}, h_j : \Omega \rightarrow \mathbb{R}$

$$\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_q(\mathbf{x})), \quad (17)$$

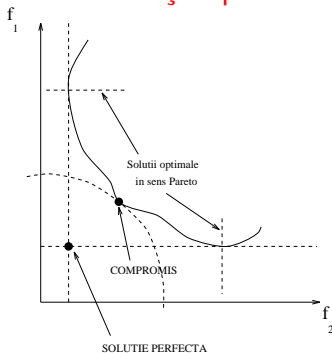
unde $f_k : \Omega \rightarrow \mathbb{R}, k = 1, q$.

De obicei obiectivele intră în conflict, soluțiile care ar minimiza fiecare obiectiv în parte sunt diferite

⇒ nu există soluție acolo unde toate obiectivele își ating minimul.

Soluții în sens Pareto

Se caută o **soluție optimală în sens Pareto**².



O problemă de optimizare în care îmbunătățirea unui obiectiv cauzează degradarea a cel puțin unui alt obiectiv nu are soluție decât în sens optimal Pareto.

Interpretarea geometrică a soluțiilor optimale în sens Pareto.

²concept introdus în 1896 pentru probleme din economie

Reformularea ca problemă de optimizare scalară

De multe ori se reduc la o problemă de optimizare scalară

- *Ponderarea obiectivelor*

$$f(\mathbf{x}) = \sum_{k=1}^q w_k f_k(\mathbf{x}) \quad (18)$$

w_k sunt ponderi care se stabilesc printr-un proces iterativ

- *Ponderarea distanțelor*

$$f(\mathbf{x}) = \sum_{k=1}^q w_k (f_k^* - f_k(\mathbf{x}))^2 \quad (19)$$

f_k^* sunt cerințele de atins (minimele funcțiilor obiectiv);

- *Folosirea unui criteriu de tip "minimax"*

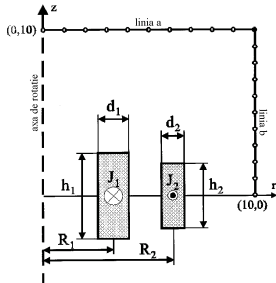
$$\min \max |w_k f_k(\mathbf{x})| \quad (20)$$

- *Reformularea problemei* - numai unul din obiective se minimizează, celelalte devin restricții suplimentare.

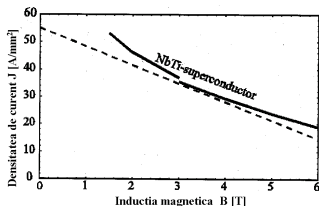
Exemplul 1

proiectare = optimizare

Ex.1. Optimizarea unui sistem de stocare a energiei (problema TEAM³)



Dispozitiv SMES cu doi solenoizi.



Restricția impusă pentru supraconductor.

³TEAM (Testing Electromagnetic Analysis Methods) = grup de lucru internațional care își propune să compare programele pentru calculul câmpului electromagnetic, detalii și formulări detaliate se găsesc la <http://www.compumag.org/jsite/team.html> nr.22)

Exemplul 1

Să se găsească $(R_1, R_2, h_1/2, h_2/2, d_1, d_2, J_1, J_2)$ având restricțiile:

	R_1 [m]	R_2 [m]	$h_1/2$ [m]	$h_2/2$ [m]	d_1 [m]	d_2 [m]	J_1 [MA/m ²]	J_2 [MA/m ²]
min	1.0	1.8	0.1	0.1	0.1	0.1	10.0	-30.0
max	4.0	5.0	1.8	1.8	0.8	0.8	30.0	-10.0

- Energia magnetică stocată să fie $E_{\text{ref}} = 180$ MJ;
- Să fie garantată supraconductibilitatea;
 $|\mathbf{J}| \leq (-6.4|\mathbf{B}| + 54.0) \text{ A/mm}^2$.
- Câmpul de dispersie (măsurat la 10 m de dispozitiv) să fie cât mai mic posibil.

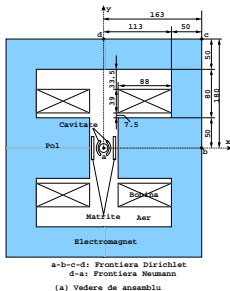
$$f(R_1, R_2, h_1/2, h_2/2, d_1, d_2, J_1, J_2) = \frac{B_{\text{stray}}^2}{B_{\text{norm}}^2} + \frac{|E - E_{\text{ref}}|}{E_{\text{ref}}} \quad (21)$$

$$E_{\text{ref}} = 180 \text{ MJ}, B_{\text{norm}} = 2.0 \cdot 10^{-4} \text{ T} \text{ și } B_{\text{stray}}^2 = \frac{\sum_{i=1}^{22} |B_{\text{stray}_i}|^2}{22}.$$

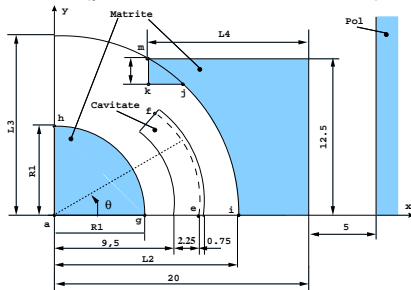
(B_{stray}^2) - folosește câmpul în puncte pe liniile a și b).

Exemplul 2

Ex.2. Optimizarea unei matrițe cu electromagnet folosită pentru orientarea pulberilor magnetice (problema TEAM nr.25)



Matriță cu electromagnet.



Detaliu în zona de interes.

Exemplul 2

Să se găsească (R_1, L_2, L_3, L_4) având restricțiile

	R_1 [mm]	L_2 [mm]	L_3 [mm]	L_4 [mm]
min	5	12.6	14	4
max	9.4	18	45	19

- a.î. pentru o solenație de 4253 A-spiră, câmpul magnetic în cavitate să fie orientat radial:

$$B_x = 0.35 \cos \theta \text{ [T]} \quad B_y = 0.35 \sin \theta \text{ [T]}$$

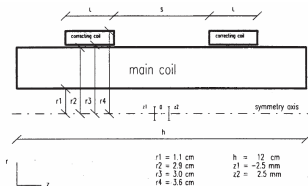
Matrița și electromagnetul au curba de magnetizare dată (oțel).

B [T]	0.0	0.11	0.18	0.28	0.35	0.74	0.82	0.91
H [A/m]	0.0	140	178	215	253	391	452	529
B [T]	0.98	1.02	1.08	1.15	1.27	1.32	1.36	1.39
H [A/m]	596	677	774	902	1164	1299	1462	1640
B [T]	1.42	1.47	1.51	1.54	1.56	1.60	1.64	1.72
H [A/m]	1851	2262	2685	3038	3395	4094	4756	7079

$$f(R_1, L_2, L_3, L_4) = \sum_{i=1}^n [(B_{xp_i} - B_{xo_i})^2 + (B_{yp_i} - B_{yo_i})^2] \quad (22)$$

Exemplul 3

Ex.3. Optimizarea unei configurații de solenoizi (problema Loney⁴)



Secțiune transversală.

Să se găsească parametrii geometrici (S, L) astfel încât câmpul magnetic în mijlocul solenoidului să fie uniform.

$$f(S, L) = \frac{B_{\max} - B_{\min}}{B_0} \quad (23)$$

⁴P. di Barba, A. Gottvald, A. Savini, *Global optimization of Loney's solenoid: a benchmark problem.*

Alte exemple - optimizări în inginerie



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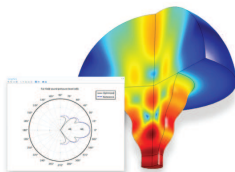


Overview

Key Features Stories Models

Optimization Module

Optimize and Improve Your Engineering Designs



A horn originally had the shape of an axisymmetric cone with a straight boundary. This is optimized with respect to the far-field sound pressure level.

Improving Your COMSOL Multiphysics Models

The Optimization Module is an add-on package that you can use in conjunction with any existing COMSOL Multiphysics Product. Once you have created a COMSOL Multiphysics model of your product or process, you always want to improve on your design. This involves four steps. First, you define your objective function - a figure of merit that describes your system. Second, you define a set of design variables - the inputs to the model that you would like to change. Third, you define a set of constraints, bounds on your design variables, or operating conditions that need to be satisfied. Last, you use the Optimization Module to improve your design by changing the design variables, while satisfying your constraints. The Optimization Module is a general interface for defining objective functions, specifying design variables, and setting up these constraints. Any model input, whether it be geometric dimensions, part shapes, material properties, or material distribution, can be treated as a design variable, and any model output can be used to define the objective function. It can be used throughout the COMSOL Multiphysics product family and can be combined with the LiveLink™ add-on products to optimize a geometric dimension in a third-party CAD program.

<http://www.comsol.com/models/optimization-module>

Alte exemple - optimizări în inginerie



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APPLICATION GALLERY


Shape Optimization of a Tuning Fork

Application ID: 8499

This model simulates a tuning fork for tuning musical instruments which, if correctly designed, should sound the note of A, 440 Hz. It computes the fundamental eigenfrequency and eigenmode for the tuning fork. Although the example seems to be somewhat academic in nature, the eigenfrequencies and eigenmodes of microscopic tuning forks are also used in quartz watches and other electronic devices.



Alte exemple - optimizări în inginerie

CST – Computer Simulation Technology 

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- CST STUDIO SUITE
 - User Interface
 - SAH
 - Optimization
 - Import/Export
 - Technical Specification
 - Recommended Hardware
- CST HWS
- CST DS
- CST EMS
- CST PS
- CST MPS
- CST PCBSS
- CST CS
- HPC
- CST EMC STUDIO
- CST BOARDCHECK
- Antenna Magus
- Other
- Publications
- Information Request

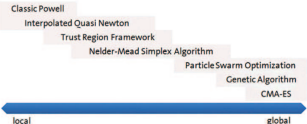
Optimization

Optimization allows engineers to get the most out of their devices. Small changes to the dimensions of a component can have a big effect on its tuning and efficiency. When multiple variables are involved, the interactions between them can be complex, and finding the optimum values analytically is often impossible.

The parameterization and optimization tools in CST STUDIO SUITE® mean that users can check how a device's behavior is affected as its properties change, and find the parameters which maximize or minimize a given effect or fulfill a certain goal. These can optimize any property of the model that can be parameterized, such as the dimensions or position of a component, the materials properties, and the values of circuit elements connected to it.

CST STUDIO SUITE contains several optimization algorithms, both local and global, each suited to different situations, as illustrated in the diagram below. **Local optimizers** provide fast convergence, but risk converging to a local minimum rather than the overall best solution. **Global optimizers** on the other hand typically require more calculations, but they search the entire problem space.

These tools can be used with any solver, both within the module itself and in CST DESIGN STUDIO™, which allows optimization tasks to be set up and run semi-automatically. For very complex systems, or problems with large numbers of variables, high-performance computing techniques are available to speed up simulation and optimization. The performance of global optimizers in particular can be greatly improved with the use of distributed computing.

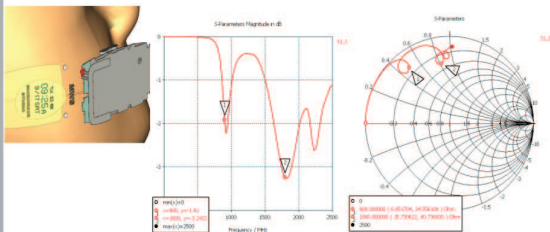


local **global**

<https://www.cst.com/Products/CSTS2/Optimization>

Alte exemple - optimizări în inginerie

Mobile Phone Antenna

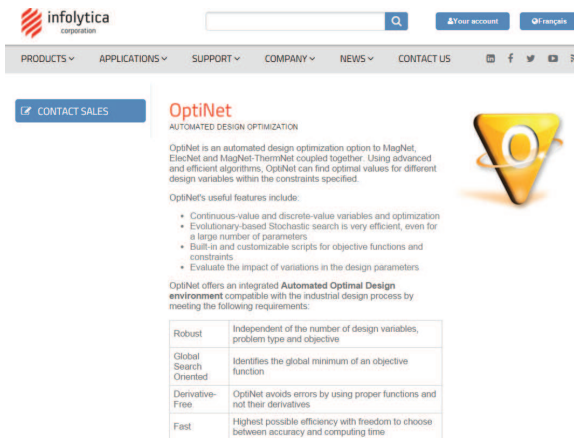


Goal: Best impedance matching in bands 890-960 MHz and 1710-1880 MHz.



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Alte exemple - optimizări în inginerie



The screenshot shows the infolytica website. At the top left is the infolytica corporation logo. To its right is a search bar and two buttons: 'Your account' and 'Français'. Below the logo is a navigation menu with links for PRODUCTS, APPLICATIONS, SUPPORT, COMPANY, NEWS, and CONTACT US. A 'CONTACT SALES' button is visible on the left. The main content area features the 'OptiNet' product title, the subtitle 'AUTOMATED DESIGN OPTIMIZATION', and a description: 'OptiNet is an automated design optimization option to MagNet, ElecNet and MagNet-ThermNet coupled together. Using advanced and efficient algorithms, OptiNet can find optimal values for different design variables within the constraints specified.' Below this is a list of features: 'OptiNet's useful features include: • Continuous-value and discrete-value variables and optimization • Evolutionary-based Stochastic search is very efficient, even for a large number of parameters • Built-in and customizable scripts for objective functions and constraints • Evaluate the impact of variations in the design parameters'. A table follows, detailing the product's characteristics. To the right of the text is a 3D yellow pyramid icon with a white 'O' and a blue ring.

OptiNet
AUTOMATED DESIGN OPTIMIZATION

OptiNet is an automated design optimization option to MagNet, ElecNet and MagNet-ThermNet coupled together. Using advanced and efficient algorithms, OptiNet can find optimal values for different design variables within the constraints specified.

OptiNet's useful features include:

- Continuous-value and discrete-value variables and optimization
- Evolutionary-based Stochastic search is very efficient, even for a large number of parameters
- Built-in and customizable scripts for objective functions and constraints
- Evaluate the impact of variations in the design parameters

OptiNet offers an integrated **Automated Optimal Design environment** compatible with the industrial design process by meeting the following requirements:

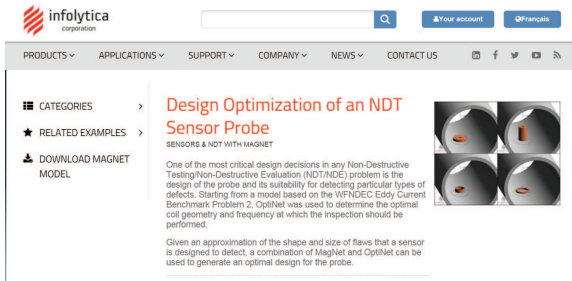
Robust	Independent of the number of design variables, problem type and objective
Global Search Oriented	Identifies the global minimum of an objective function
Derivative-Free	OptiNet avoids errors by using proper functions and not their derivatives
Fast	Highest possible efficiency with freedom to choose between accuracy and computing time

Alte exemple - optimizări în inginerie

The screenshot shows the Infolytica website interface. At the top left is the Infolytica logo. To its right is a search bar and two buttons: 'Your account' and 'Français'. Below the logo is a navigation menu with items: PRODUCTS, APPLICATIONS, SUPPORT, COMPANY, NEWS, and CONTACT US. On the left side, there is a sidebar with 'CATEGORIES', 'RELATED EXAMPLES', 'DOWNLOAD MAGNET MODEL', and 'VIDEOS'. The main content area features an article titled 'Advanced Optimization of an IPM Machine' with the subtitle 'MOTORS AND GENERATORS WITH MAGNET'. The article text describes the optimization of a 3-phase, 4-pole single-barrier IPM motor. To the right of the text is a 3D CAD model of a motor's cross-section. Below the article text is a smaller image of the motor's cross-section.

<http://www.infolytica.com/en/applications/ex0123/>

Alte exemple - optimizări în inginerie



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Design Optimization of an NDT Sensor Probe

SENSORS & NDT WITH MAGNET

One of the most critical design decisions in any Non-Destructive Testing/Non-Destructive Evaluation (NDT/NDE) problem is the design of the probe and its suitability for detecting particular types of defects. Starting from a model based on the WFNDEC Eddy Current Benchmark Problem 2, OptiNet was used to determine the optimal coil geometry and frequency at which the inspection should be performed.

Given an approximation of the shape and size of flaws that a sensor is designed to detect, a combination of MagNet and OptiNet can be used to generate an optimal design for the probe.

<http://www.infolytica.com/en/applications/ex0116/>

Alte exemple - optimizări în inginerie

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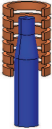
📄 DOWNLOAD MAGNET MODEL

Coil Size Optimization - Induction Heating

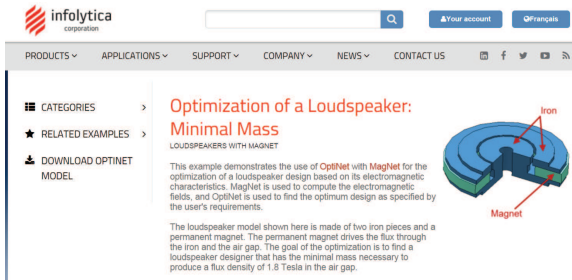
INDUCTION HEATING WITH MAGNET

In the multiple-coil configuration shown in this figure, the work piece is surrounded by six coils (coils are shown partially so that the workpiece can be seen). The objective of this optimization is to find the inner radii of the coils in order to obtain a uniform temperature in the upper portion of the workpiece.

The coupled electromagnetic-thermal simulation is a transient thermal solution that, at each time step during the transient process, performs a time-harmonic electromagnetic solution to update the eddy current losses. The workpiece is made of stainless steel and its material properties are non-linear and vary with temperature.



Alte exemple - optimizări în inginerie



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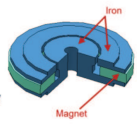
📄 DOWNLOAD OPTINET MODEL

Optimization of a Loudspeaker: Minimal Mass

LOUDSPEAKERS WITH MAGNET

This example demonstrates the use of **OptiNet** with **MagNet** for the optimization of a loudspeaker design based on its electromagnetic characteristics. MagNet is used to compute the electromagnetic fields, and OptiNet is used to find the optimum design as specified by the user's requirements.

The loudspeaker model shown here is made of two iron pieces and a permanent magnet. The permanent magnet drives the flux through the iron and the air gap. The goal of the optimization is to find a loudspeaker designer that has the minimal mass necessary to produce a flux density of 1.8 Tesla in the air gap.



<http://www.infolytica.com/en/applications/ex0086/>

Alte exemple - optimizări în inginerie

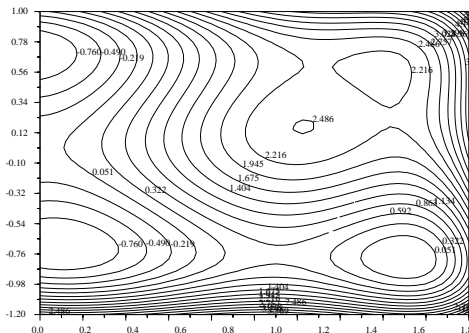
The screenshot shows the website for infolytica corporation. The header includes the company logo, a search bar, and navigation links for 'Your account' and 'Français'. Below the header is a main navigation menu with categories like PRODUCTS, APPLICATIONS, SUPPORT, COMPANY, NEWS, and CONTACT US. The main content area features a sidebar with 'CATEGORIES' and 'RELATED EXAMPLES', and a main article titled 'SRM Design Optimization' with the subtitle 'MOTORS & GENERATORS WITH MAGNET'. The article text describes the complexity of SRM design and mentions the use of OptiNet. To the right of the text is a circular diagram of an 8/6 SRM stator and rotor assembly.

<http://www.infolytica.com/en/applications/ex0132/>

Exemple simple pentru testarea algoritmilor

Funcția "six-hump camel back" (cămila cu șase cocoașe)

$$C(x, y) = \left(4 - 2.1x^2 + \frac{x^4}{3}\right)x^2 + xy + (-4 + 4y^2)y^2 \quad (24)$$



$$-3 \leq x \leq 3 \text{ și}$$

$$-2 \leq y \leq 2$$

Un minim global = -1.03163

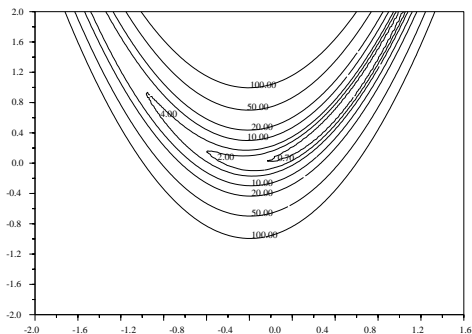
în două puncte diferite: $(x, y) = (-0.0898, -0.7126)$

și $(0.0898, -0.7126)$.

Exemple simple pentru testarea algoritmilor

Funcția lui Rosenbrock ("funcția banană")

$$B(x, y) = 100(y - x^2)^2 + (1 - x)^2 \quad (25)$$



$$-2.048 \leq x \leq 2.048 \text{ și}$$

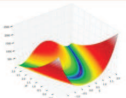
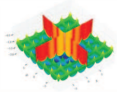
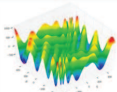
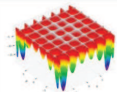
$$-2.048 \leq y \leq 2.048$$

Un minim global
 egal cu 0 în punctul
 $(x, y) = (1, 1)$.

Minime locale nu
 există, dar funcția
 are un relief com-
 plicat pentru algo-
 ritmii de optimizare

Exemple simple pentru testarea algoritmilor

Alte exemple de acest tip https://en.wikipedia.org/wiki/Test_functions_for_optimization

Rosenbrock function		Cross-in-tray function:		$f(x, y) = -0.0001 \left(\left \sin(x) \sin(y) \exp \left(\left 100 - \frac{\sqrt{x^2 + y^2}}{\pi} \right \right) + 1 \right \right)^{0.1}$
		Eggholder function:		$f(x, y) = -(y + 47) \sin \left(\sqrt{\left y + \frac{x}{2} + 47 \right } \right) - x \sin \left(\sqrt{\left x - (y + 47) \right } \right)$
		Hölder table function:		$f(x, y) = - \left \sin(x) \cos(y) \exp \left(\left 1 - \frac{\sqrt{x^2 + y^2}}{\pi} \right \right) \right $

Clasificarea metodelor

În general, metodele de optimizare sunt iterative.

$$E_0, E_1, \dots, E_k, \dots$$

E_k este valoarea minimă obținută la iterația k .

- Dacă E_k nu scade un număr de iterații, este posibil să se fi atins un minim, dar este imposibil să se precizeze dacă acesta este global sau nu.
- Nicio tehnică de optimizare nu garantează atingerea unui minim global.

Viteza relativă de convergență = se estimează de obicei prin numărul de evaluări ale funcției obiectiv necesare pentru a reduce valoarea E_k de un anumit număr de ori.

Metode deterministe

I. Deterministe - conduc la aceeași soluție pentru rulări diferite ale programului, dacă pornesc din aceleași condiții inițiale și au aceleași parametri.

- Dezavantaj: găsesc întotdeauna un minim local, dependent de inițializare;
- Avantaj: efort de calcul mic.

În problemele de optimizare din efortul de calcul se exprimă în număr de evaluări de funcții obiectiv.

Pot fi

- 1 **de ordin zero** - necesită doar evaluări de funcții obiectiv;

Ex: metoda căutării simultane; metoda căutării dihotomice; metoda Fibonacci; metoda secțiunii de aur; metoda simplexului descendent (Nelder-Mead); metoda Powell, etc.

- 2 **de ordin superior** (1,2) - necesită și evaluări ale derivatelor funcției obiectiv.

Metode stocastice

II. Stocastice - au un caracter aleator, nu conduc la aceeași soluție, chiar dacă pornesc din aceleași condiții inițiale și au aceleași parametri;

- Dezavantaj: necesita un efort de calcul foarte mare.
- Avantaj: au o probabilitate foarte mare de a găsi un minim global.

Ex: metoda căutării aleatoare; algoritmi evoluționiști; algoritmi genetici, optimizare bazată pe roiuri de particule (*particle swarm*), colonii de furnici (*ant colony*), călirea simulată (*simulated annealing*), căutare tabu (*tabu search*), etc.

Răsfoiți și https://en.wikipedia.org/wiki/Mathematical_optimization

- [Ciuprina02] G.Ciuprina, D.Ioan, I.Munteanu, M.Rebican, R.Popa, Optimizarea numerica a dispozitivelor electromagnetice, Editura Printech, 2002.

disponibilă la <http://www.lmn.pub.ro/~gabriela/books/opt2002.pdf>

- [Cheney08] Ward Cheney and David Kincaid, *Numerical Mathematics and Computing*, Brooks/Cole publishing Company,2008. (Capitolul 16 - Minimization of functions)